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A fixed point theorem for expansive type mapping in dislocated Quasi-metric space

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ABSTRACT : In this paper we have proved a fixed point theorem for continuous surjective mapping in dislocated quasi metric space.

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I. INTRODUCTION AND PRELIMINARIES

The studies of fixed point on dislocated metric space have attracted much attention, some of the literatures may be noted in [1, 2, 3, 4, 5, 6]. In this paper we construct a sequence and consider its convergence to the unique fixed point of a self map.

Definition 1. [5] Let X be a nonempty set and let $d : X \times X \to (0, \infty)$ be a function satisfying following conditions:

(i) $d(x, y) = d(y, x) =$ implies $x = y$,

(ii) $d(x, y) \le d(x, z) + d(z, y)$ For all $x, y, z \in X$,

Then d is called a dislocated quasi-metric on X. If d satisfies $d(x, y) = d(y, x)$, then it is called dislocated metric.

Definition 2. [5] A sequence $\{x_n\}$ in dq-metric space (dislocated quasi-metric space) (X, d) is called Cauchy sequence if for given $\varepsilon > 0$, $\exists\ n_0 \in N$, such that $\forall\ m, n \ge n_0$, implies $d(x_m, x_n) < \varepsilon$ or $d(x_n, x_m) < \varepsilon$ *i.e.* $\min\{d(x_m, x_n), d(x_n, x_m)\} < \varepsilon$.

Defintion 3. [5] A sequence $\{x_n\}$ dislocated quasi-converges to x if

$$\lim_{n\to\infty} (x_n, x) = \lim_{n\to\infty} (x, x_n) = 0$$

In this case x is called a dq-limit of $\{x_n\}$ and we write $x_n \to x$.

Leema 4. [5] dq-limits in a dq-metric space are unique.

Definition 5. [5] A dq-metric space (X, d) is called complete if every Cauchy sequence in it is dq-convergent.

Definition 6. [5] Let (X, d_1) and (Y, d_2)be a dq-metric spaces and let $f : X \to Y$ be a function. Then f is continuous to $x_0 \in X$, if for each sequence $\{x_n\}$ which is $d_1 - q$ convergent to x_0, the sequence $\{f(x_n)\}$ is $d_2 - q$ convergent to $f(x_0)$ in Y.

Definition 7. [5] Let (X, d) be a dq-metric space. A map $T : X \to X$ is called contraction if therese exists $0 \le K < 1$ such that

$$d(Tx, Ty) \le Kd(x, y) \qquad \text{for all } x, y \in X.$$

Definition 8. A function $f : X \to Y$ is surjective if and only if for every y in the co domain of Y. There is at least one x in domain X such that $f(x) = y$.

Theorem 9. [5] Let (X, d) be a dq-metric space and let $T : X \to X$ be a continuous. Then T has unique fixed point.

II. MAIN RESULT

Theorem 1. Let (X, d) be a complete dq-metric space and let $T : X \to X$ be a surjective continuous mappings satisfying the follows condition.

$$d(Tx, Ty) \ge \frac{\alpha[1 + d(Ty, y)]\, d(Tx, x)}{1 + d(x, y)} + \beta\, d(x, y \qquad \text{...(3.1)}$$

for all $x, y \in X$, $\alpha, \beta > 0$ and $\alpha + \beta > 1$. Then T has a unique fixed point. If further $\beta > 1$ then this fixed be unique.

Proof. Let $\{x_n\}$ be a sequence in X defined as follows.
Let $x_0 \in X$, $T(x_1) = x_0$, $T(x_2) = x_1$, ...,$T(x_{n+1}) = x_n$ ....
Consider

$$d(x_{n-1}, x_n) = d(Tx_n, Tx_{n+1}) \ge \frac{\alpha[1 + d(Tx_{n+1}, x_{n+1})]\, d(Tx_n, x_n)}{1 + d(x_n, x_{n+1})} + \beta\, d(x_n, x_{n+1})$$

$$= \frac{\alpha[1 + d(x_n, x_{n+1})]\, d(x_{n-1}, x_n)}{1 + d(x_n, x_{n+1})} + \beta\, d(x_n, x_{n+1})$$

$$\ge \alpha\, d(x_{n-1}, x_n) + \beta\, d(x_n, x_{n+1})$$

Therefore,

$$d(x_n, x_{n+1}) \le \frac{1-\alpha}{\beta}\, d(x_{n-1}, x_n)$$

$$= Kd(x_{n-1}, x_n)$$

where $K = \dfrac{1-\alpha}{\beta}$ with $0 \le K \le 1$. Similarly, we show that

$$d((x_{n-1}, x_n) \le Kd(x_{n-2}, x_{n-1})$$

and

$$d((x_n, x_{n+1}) \le K^2\, d(x_{n-2}, x_{n-1})$$

Thus

$$d(x, x_{n+1}) \le Kd((x_1, x_0) \qquad \text{...(3.2)}$$

$$d(x, x_{n+1}) < K^n d(x_1, x_0)$$

Since $0 \le K < 1$, as $n \to \infty$, $K^n \to 0$. Hence $\{x_n\}$ is a dq-sequence in the complete dq-metric space X. Thus $\{x_n\}$ dislocated quasi converges to some $x \in X$.

Existence of a fixed point

Since T is a surjective map then there exist a point y in X such that

$$x = Ty. \quad ...(3.3)$$

Consider

$$d(x_n, x) = d(Tx_{n+1}, Ty) \geq \frac{\alpha[1 + d(Ty, y)]\, d(Tx_{n+1}, x_{n+1})}{1 + d(x_{n+1}, y)} + \beta d(x_{n+1}, y)$$

$$= \frac{\alpha[1 + d(x, y)]\, d(x_n, x_{n+1})}{1 + d(x_{n+1}, y)} + \beta d(x_{n+1}, y)$$

Since $\{x_{n+1}\}$ is a subsequence of $\{x_n\}$ and $\{x_n\}$ dislocated quasi convergs to x.

$$\Rightarrow \{x_{n+1}\} \to x \text{ when } n \to \infty$$

$$d(x, x) \geq \frac{\alpha[1 + d(x, y)]\, d(x, x)}{1 + d(x, y)} + \beta d(x, y)$$

$$\Rightarrow 0 \geq 0 + \beta d(x, y)$$

$$\Rightarrow \beta d(x, y) \leq 0 \qquad [As\ \beta > 0]$$

$$\Rightarrow d(x, y) = 0$$

$$\Rightarrow x = y$$

$\therefore$ From equation (3.3) we have $x = Tx$

Thus T has a fixed point.

Uniqueness. Let u be another fixed point of T in X *i.e.* $Tu = u$

Now

$$d(x, u) = d(Tx, Tu) \geq \frac{\alpha[1 + d(Tu, u)]\, d(Tx, x)}{1 + d(x, u)} + \beta d(x, u)$$

$$= \frac{\alpha[1 + d(u, u)]\, d(x, x)}{1 + d(x, u)} + \beta d(x, u)$$

$$\Rightarrow d(x, u) \geq \beta d(x, u)$$

$$\Rightarrow (1 - \beta)\, d(x, u) \geq 0$$

$$\Rightarrow d(x, u) = 0 \qquad [As\ \beta > 1.]$$

$$\Rightarrow x = u$$

Thus fixed point of T is unique.

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